The central scientific problem of this project was to factor a matrix $Y$ (the observed data) into the product of two other matrices, say $Y = MX$, where each row of $X$ corresponds to a source and $M$ is the so-called mixing matrix. We developed methods for the inference of $M$ and $X$ in two applications: unmixing of phase-synchronous signals and of cardiac signals. The inference of $M$ and $X$ is an inverse problem, which was addressed under the regularization theory framework.

Matrix factorization is a longstanding problem that comes under many different names, depending on the assumptions taken for $M$ and $X$. Some popular terms are blind source separation (BSS), independent component analysis (ICA), dependent component analysis, and non-negative matrix factorization (NMF).

Without any constraint or assumption on $M$ and $X$, the matrix factorization problem is ill-posed, since it has a very large set of solutions, even after factoring out permutations, scalings, and reversed signs of the sources. It happens, however, that each application has its own set of constraints and assumptions concerning $M$ and $X$ and also about possible perturbations to $Y$ due, e.g., to noise. These constraints and assumptions narrow down the set of solutions of the matrix factorization problem.

The most widely used assumption in BSS is that sources are statistically independent. This rather simple assumption is the thrust underlying a vast variety of BSS approaches, often termed independent component analysis. However, in the problems addressed in this project, the sources are not independent. Therefore, the use of ICA concepts and algorithms is questionable, as shown in various works.

A considerable research effort has been devoted, in the last decade, to BSS with statistically dependent sources. The general trend has been to develop specific solutions that fully exploit the constraints and assumptions inherent to each application. This was the line of attack followed in this project. In summary, we approached both applications under the regularization theory by minimizing an objective function containing one term accounting for the data misfit $YMX$ and regularizer terms accounting for desired properties of the sources $X$.

In the case of synchronous signals, the regularizers promote synchrony and in the case of cardiac signals, the regularizers promote sparsity in the time-frequency domain.

The main challenge in the above formulation is the nonconvex nature of the resulting optimization. We tackled this difficulty by exploiting specific constraints linked with each application and by using recent alternating optimization results in the area of non-convex optimization. The developed methodologies were successfully applied to simulated signals, to magnetoencephalographic (MEG) signal, to heart signals.

Source separation of synchronous signals

$$y(t) = Mx(t)$$

where $y(t)$ is the observation vector, $x(t)$ is the source vector, and $M$ is the mixing matrix.

The $i$th component of the observation vector $y(t)$ is given by

$$y_i(t) = \sum_j M_{ij} x_j(t)$$

with $M_{ij}$ being the element of $M$ in the $i$th row and $j$th column.

$$x_i(t) = \sum_j M_{ij}^{-1} y_j(t)$$

where $M_{ij}^{-1}$ is the element of $M^{-1}$ in the $i$th row and $j$th column.

Cardiac signals:

- Denoising methodology for heart signals using matching pursuit.

- BSS methodology for periodic sources from sequentially recorded instantaneous mixtures.

- Segmentation methodology based on wavelets for the of heart signals from pediatric auscultations.

All the methodologies above mentioned include inference criteria, optimization algorithms, and code.